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Technical Report 32-1141

Acoustic Spectrum Shaping Utilizing Finite Hyperbolic Horn Theory

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Abstract

This report describes the technique developed for shaping the sound spectrum of high intensity sound to achieve specialized response characteristics from single horns or multiple horn arrays. The design parameter analysis for the technique developed includes computations and computer simulation of horn responses using the hyperbolic horn theory. This technique provides the ability to minimize any undertesting or overtesting of the test article.

Acoustic Spectrum Shaping Utilizing Finite Hyperbolic Horn Theory

I. Introduction

This study investigates a technique for shaping the sound spectrum of high intensity sound with relation to better acoustic environmental testing. The computations and computer simulations used in this study are based on high-intensity sound generator systems.

The frequency response characteristics of an acoustic noise generating system are dependent upon the noise generator, acoustic horn (a coupler), and the region for which the noise is intended (such as a test chamber, progressive wave tube, or free space, etc.). Each of these elements has separate frequency response characteristics, therefore, the frequency response characteristics of the entire system are a function of these characteristics.

At the present time, developmental contracts through other government and NASA centers are developing high-intensity sound generators with broader frequency response characteristics and greater sound power capabilities. All these developmental studies deal strictly with the sound generator. This particular study is unique in that the sound generator remains constant and the sound spectrums are varied by specially designed acoustic horns.

The design for the acoustic horns is based on acoustic horn theory for a special family of horns, which are referred to as *Hyperbolic Horns*. These horns have, theoretically, the unique property of a resonant response characteristic at frequencies just above their cutoff frequency.

This study includes a complete mathematical analysis for hyperbolic horns, and a digital computer program to obtain frequency response data for a given single horn with particular response characteristics. From this study, a technique is developed for designing multiple horn arrays. This technique consists of designing each horn of the array slightly different from the other horns so that the next response function for the array has a smoother energy spectrum than any of the individual horns. The smoother spectrum is obtained by effectively averaging the outputs from all of the individual horns.

The techniques developed in this study will allow a design parameter analysis for both single horns and multiple horn arrays, with optimized response characteristics of the single horn (or of an entire multiple horn array).

The ability to more accurately simulate broadband acoustic spectrums for reliable acoustic testing is extremely important to minimize the probability of undertesting or overtesting the test article. These design techniques can also be applied in the audio engineering field, i.e., sound reproduction systems could be made better by improving the response characteristics of the system loudspeakers, both individually and as an array.

II. Hyperbolic Horn Analysis

Basic horn theory for a hyperbolic horn of infinite length predicts a very sharp frequency cutoff, depending on the horn flare constant and on the velocity of sound in the medium which for this study is air. For a hyperbolic horn of finite length, the response characteristics as a function of frequency are dependent upon the following:

(1) Flare constant N.

(2) Family parameter T.

Note: The value $T = 1$ in hyperbolic horn equation theory results in the exponential horn.

(3) Length L.

(4) Termination condition z_M at the mouth of the horn, i.e., whether it opens into free space or an infinite progressive wave tube, etc.

(5) Velocity of sound in the medium (air).

Expressions for the hyperbolic horn response characteristics are given in Refs. 1 and 2. These expressions are derived for frequencies above the cutoff frequency; however, these expressions must be modified to correctly describe the characteristics at the cutoff frequency and below the cutoff frequency. These response characteristics are given in the form of impedance expressions; therefore, the effect of the horn on the transmission of acoustic energy can be described in terms of a lumped impedance in an equivalent electrical analog circuit.

A. Summary of Equations for z_T

Derivations of the required expressions are included in Appendix A. Starting with the above cutoff expression for the acoustic impedance z_T of the general hyperbolic horn, the following expressions for z_T are obtained:

(1) For the general hyperbolic horn.

(a) z_T at cutoff.

(b) z_T below cutoff.

(2) For the special case $T = 1$, hyperbolic horn (the exponential horn).

(a) z_T above cutoff.

(b) z_T at cutoff.

(c) z_T below cutoff.

The following is a summary of these expressions for z_T, including the above cutoff expression for the general hyperbolic horn.

1. General hyperbolic horn

Above cutoff ($f > f_c$):

$$z_T = \frac{1}{T}\left\{\frac{\left[\left(\frac{jk - NTz_M}{b}\right) + \left(\frac{jkT - Nz_M}{b}\right)\tanh(NL)\right]\sin(bL) + z_M\left[1 + T\tanh(NL)\right]\cos(bL)}{\left[\frac{1}{b}\left\{N - \left(\frac{b^2 + N^2T^2}{jkT}\right)z_M\right\} + \left(\frac{NT + jkz_M}{b}\right)\tanh(NL)\right]\sin(bL) + \left[\frac{1}{T} + \left\{1 - \frac{N(1 - T^2)z_M}{jkT}\right\}\tanh(NL)\right]\cos(bL)}\right\} \tag{1}$$

At cutoff ($f = f_c$):

$$z_T = \left\{N\frac{\left[-LNz_M\{T + \tanh(NL)\} + z_M\{1 + T\tanh(NL)\}\right] + j\left[NL\{1 + T\tanh(NL)\}\right]}{\left[LN^2T\{1 + T\tanh(NL)\} + N\{1 + T\tanh(NL)\}\right] + j\left[LN^2T^2z_M + \{LN^2Tz_M + N(1 - T^2)\}\tanh(NL)\right]}\right\} \tag{2}$$

Below cutoff ($f < f_c$):

$$z_T = k\left\{\frac{[k\{1 + T\tanh(NL)\}\sinh(BL)]}{[\{(-B^2 + N^2T^2)z_M + k^2Tz_M\tanh(NL)\}\sinh(BL) + \{BN(1-T^2)z_M\tanh(NL)\}\cosh(BL)]}\right.$$

$$\left.\frac{+j[Nz_M\{T + \tanh(NL)\}\sinh(BL) - (Bz_M)\{1 + T\tanh(NL)\}\cosh(BL)]}{-j[NkT\{1 + T\tanh(NL)\}\sinh(BL) + Bk\{1 + T\tanh(NL)\}\cosh(BL)]}\right\} \quad (3)$$

2. *Special hyperbolic horn (T = 1, exponential horn)*

Above cutoff:

$$z_T = \left\{\frac{z_M[\cos(bL + \theta)] + j[\sin(bL)]}{jz_M[\sin(bL)] + [\cos(bL - \theta)]}\right\} \quad (4)$$

At cutoff:

$$z_T = \left[\frac{z_M\left(1 - \frac{LM}{2}\right) + j\left(\frac{ML}{2}\right)}{\left(1 + \frac{ML}{2}\right) + jz_M\left(\frac{LM}{2}\right)}\right] \quad (5)$$

Below cutoff:

$$z_T = \left\{\frac{\left[\sinh(BL) - j\frac{z_M}{(N^2 - B^2)^{1/2}}\{B\cosh(BL) - N\sinh(BL)\}\right]}{\left[z_M\sinh(BL) - j\frac{1}{(N^2 - B^2)^{1/2}}\{B\cosh(BL) + N\sinh(BL)\}\right]}\right\} \quad (6)$$

All the expressions for z_T can be written in the form:

$$z_T(\omega) = R_T(\omega) + jX_T(\omega) \quad (7)$$

where

$R_T(\omega)$ = resistive component of impedance, and represents energy transfer by horn

and

$X_T(\omega)$ = reactive component of acoustic impedance.

III. Horn Design

Horns with particular response characteristics can now be designed, using the results of the previous hyperbolic horn analysis of paragraph II and the resulting expressions for the acoustic impedance z_T at the throat of a given horn.

The physical parameters of a particular hyperbolic horn are determined from the following equation:

$$S_x = S_T[\cosh(Nx) + T\sinh(Nx)]^2 \quad (8)$$

Equation (8) may be written in the exponential form as

$$S_x = S_T\left[\frac{(e^{Nx} + e^{-Nx})}{2} + T\frac{(e^{Nx} - e^{-Nx})}{2}\right]^2 \quad (9)$$

where

$$\cosh(Nx) = \frac{e^{Nx} + e^{-Nx}}{2}$$

$$\sinh(Nx) = \frac{e^{Nx} - e^{-Nx}}{2}$$

Equation (9) may be rewritten

$$e^{Nx}(1 + T) + e^{-Nx}(1 - T) - 2(S_x/S_T)^{1/2} = 0 \quad (10)$$

or

$$e^{2Nx}(1 + T) - 2(S_x/S_T)^{1/2}e^{Nx} + (1 - T) = P(x) = 0 \quad (11)$$

the length L of a particular hyperbolic horn is determined from Eq. (11) as follows:

$$P(x)_{x=L} = e^{2NL}(1 + T) - 2(S_M/S_T)^{1/2}e^{NL} + (1 - T) = 0 \quad (12)$$

Note: $S_L = S_{M(\text{Mouth})}$

letting

$$e^{NL} = \psi$$

Therefore,

$$L = (1/N)\log_e \psi \tag{13}$$

where ψ is found from Eq. (10), written as:

$$\psi^2(1+T) - 2(S_M/S_T)^{1/2}\psi + (1-T) = 0 \tag{14}$$

Therefore,

$$\psi = \frac{(S_M/S_T)^{1/2} \pm [(S_M/S_T) - (1-T^2)]^{1/2}}{(1+T)} \tag{15}$$

For the physical case of the length L of the horn, the root is used:

$$\psi = \frac{(S_M/S_T)^{1/2} + [S_M/S_T) - (1-T^2)]^{1/2}}{(1+T)} \tag{16}$$

When the cross sectional area at the throat S_T of the horn and the cross sectional area at the mouth S_M of the horn are defined, the length L is then only a function of T and N, or

$$L = L(N,T) = (1/N)\log_e \left\{\frac{D + [D^2 - (1-T^2)]^{1/2}}{(1+T)}\right\} \tag{17}$$

where

$$D = (S_M/S_T)^{1/2}$$

The value of N is determined from the cutoff frequency f_c of the horn, using the following expression:

$$N = (k)_{f=f_c} = \frac{2\pi f_c}{c} \tag{18}$$

Equation (17) may now be written, in terms of the cutoff frequency, as:

$$L = \left(\frac{c}{2\pi f_c}\right)\log_e \left\{\frac{D + [D^2 - (1-T^2)]^{1/2}}{(1+T)}\right\} \tag{19}$$

Equation (19) is, therefore, the expression for the length L of a given horn in terms of:

(1) The cutoff frequency f_c.

(2) The speed of sound c.

(3) The square root of ratio of the cross sectional area at the mouth of the horn to the cross sectional area at the throat $(S_M/S_T)^{1/2}$.

(4) The shape parameter T.

Figure 1 shows an acoustic horn designed to these specifications. The equations, shown in Fig. 1, used in the design of the horn are:

$$y(x) = y_T[\cosh(Nx) + T\sinh(Nx)] \tag{20}$$

$$y(x) = -y_T[\cosh(Nx) + T\sinh(Nx)] \tag{21}$$

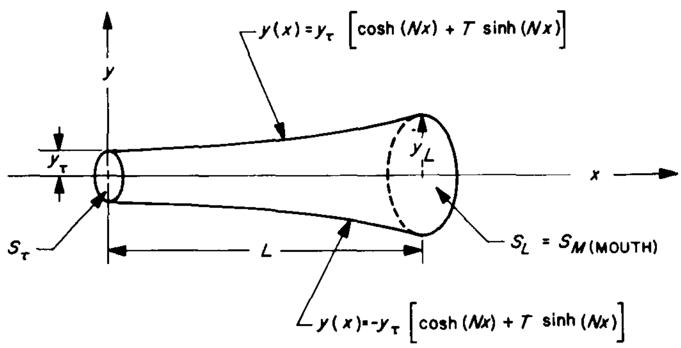


Fig. 1. Acoustic horn diagram

and are derived from the following:

$$S_x = S_T[\cosh(Nx) + T\sinh(Nx)]^2 \tag{22}$$

where

$$S_x = \begin{cases} \pi y^2(x) \text{ (for circular cross section)} \\ 4y^2(x) \text{ (for square cross section)} \\ \gamma y^2(x) \text{ (for generalized cross section)} \end{cases}$$

$$S_T = \begin{cases} \pi y_T^2 \text{ (for circular cross section)} \\ 4y_T^2 \text{ (for square cross section)} \\ \gamma y_T^2 \text{ (for generalized cross section)} \end{cases}$$

$$S_L = S_{M(\text{Mouth})} = \begin{cases} \pi y_L^2 \text{ (for circular cross section)} \\ 4y_L^2 \text{ (for square cross section)} \\ \gamma y_L^2 \text{ (for generalized cross section)} \end{cases}$$

Therefore,

$$\gamma y^2(x) = \gamma y_T^2[\cosh(Nx) + T\sinh(Nx)]^2 \tag{23}$$

or

$$y(x) = \pm y_T[\cosh(Nx) + T\sinh(Nx)] \tag{24}$$

The hyperbolic horn data used for this parametric study are given in Table 1. The digital computer program written to perform this study using this data is given in Appendix B. This program is written for one particular group of horns in which the cutoff frequencies f_c were 25, 50, and 100 Hz. Typical curves of the predicted horn responses are shown in Fig. 2. The produced curves graphically illustrate the computed response of the resistive component $R(\omega)$ for the acoustic impedance versus frequency f in relation to the hyperbolic horn family parameter T and a fixed ratio of $(S_M/S_T)^{1/2}$. This fixed ratio is:

$$\left(\frac{S_M}{S_T}\right)^{1/2} = \left(\frac{49.00 \text{ in.}^2}{1.563 \text{ in.}^2}\right)^{1/2} = D = 5.585. \qquad (25)$$

Table 1. Hyperbolic horn data used in parametric study

Horn	T	$L(T) \sim$ m	Horn	T	$L(T) \sim$ m
1	0.001	1.3162	14	3.00	0.5944
2	0.010	1.3113	15	4.00	0.4964
3	0.100	1.2646	16	5.00	0.4239
4	0.200	1.2171	17	6.00	0.3685
5	0.300	1.1735	18	7.00	0.3250
6	0.400	1.1333	19	8.00	0.2902
7	0.500	1.0959	20	9.00	0.2617
8	0.600	1.0611	21	10.00	0.2382
9	0.700	1.0285	22	20.00	0.1238
10	0.800	0.9978	23	50.00	0.0501
11	0.900	0.9690	24	100.00	0.0251
12	1.00	0.9417	25	500.00	0.0050
13	2.00	0.7325	26	1000.00	0.0025
			27	10,000.00	0.0003

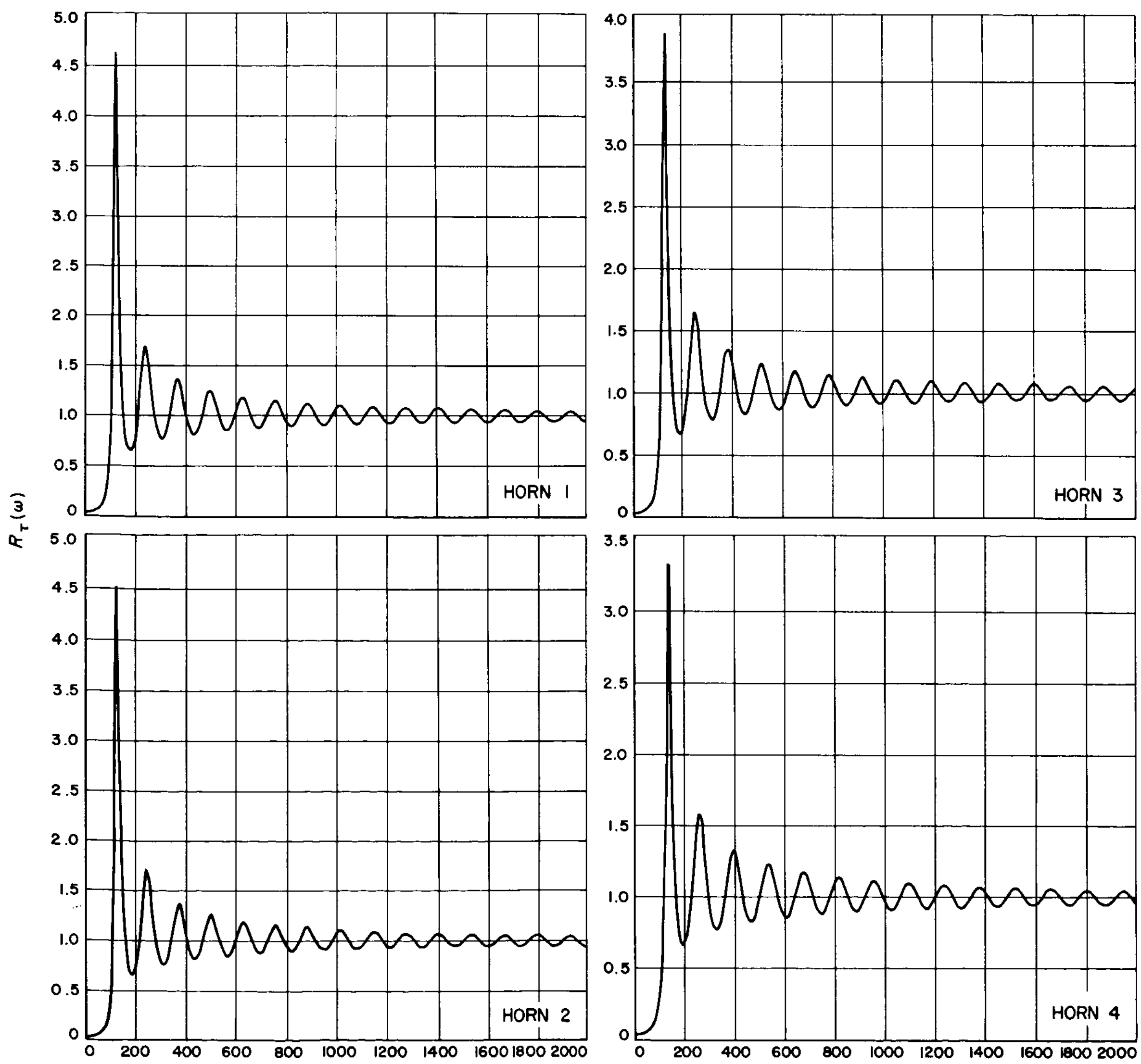


Fig. 2. T-parameter study of hyperbolic horns (frequency response characteristics)

The cutoff frequency f_c is held constant at 100 Hz, therefore,

$$L = \left(\frac{c}{2\pi f_c}\right) \log_e \left\{\frac{D + [D^2 - (1 - T^2)]^{1/2}}{(1 + T)}\right\} \qquad (26)$$

where

$$f_c = 100 \text{ Hz}$$

and

$$D = 5.585.$$

Equation (23) may be written as a function of only L as:

$$L = L(T) = 0.547 \log_e \left\{\frac{(5.585) + [(5.585)^2 - (1 - T^2)]^{1/2}}{(1 + T)}\right\} \qquad (27)$$

A ρc termination Z_M at the mouth of the horn is used for the study. This termination is necessary to reveal the characteristics of the horn, independent of its particular termination. Naturally, the nature of Z_M (i.e., zero baffle, infinite baffle, etc.) greatly affects Z_T for a particular configuration.

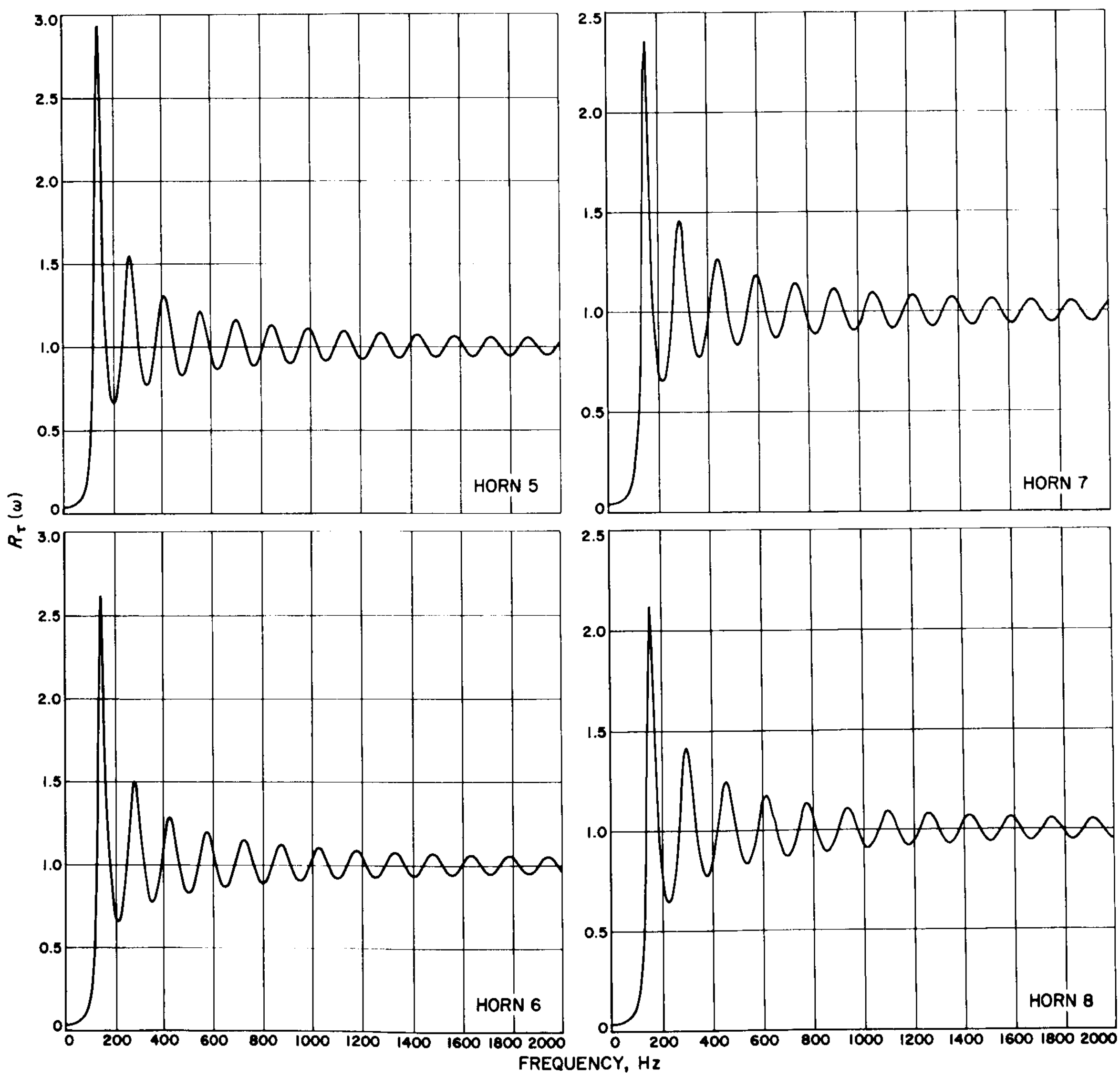


Fig. 2 (contd)

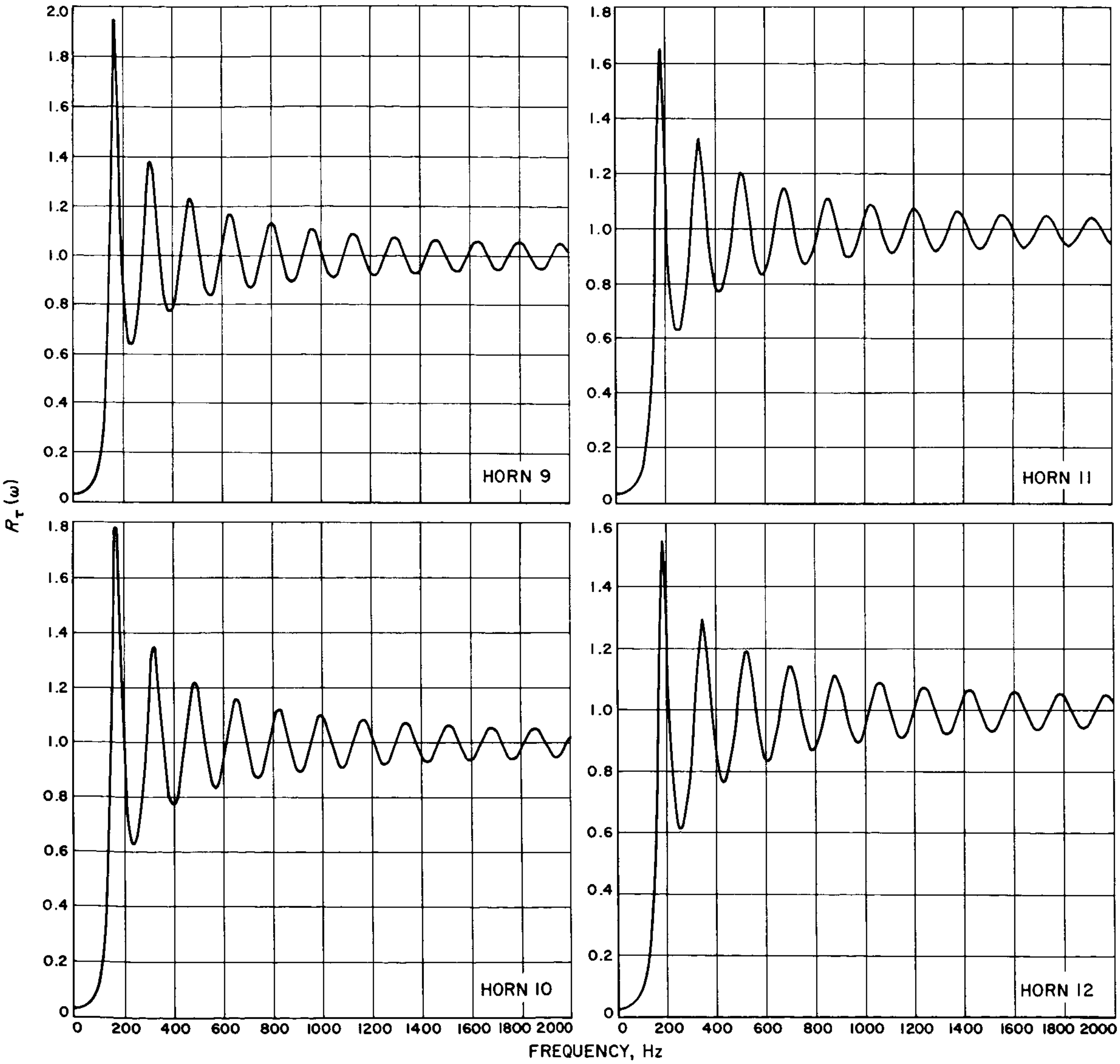
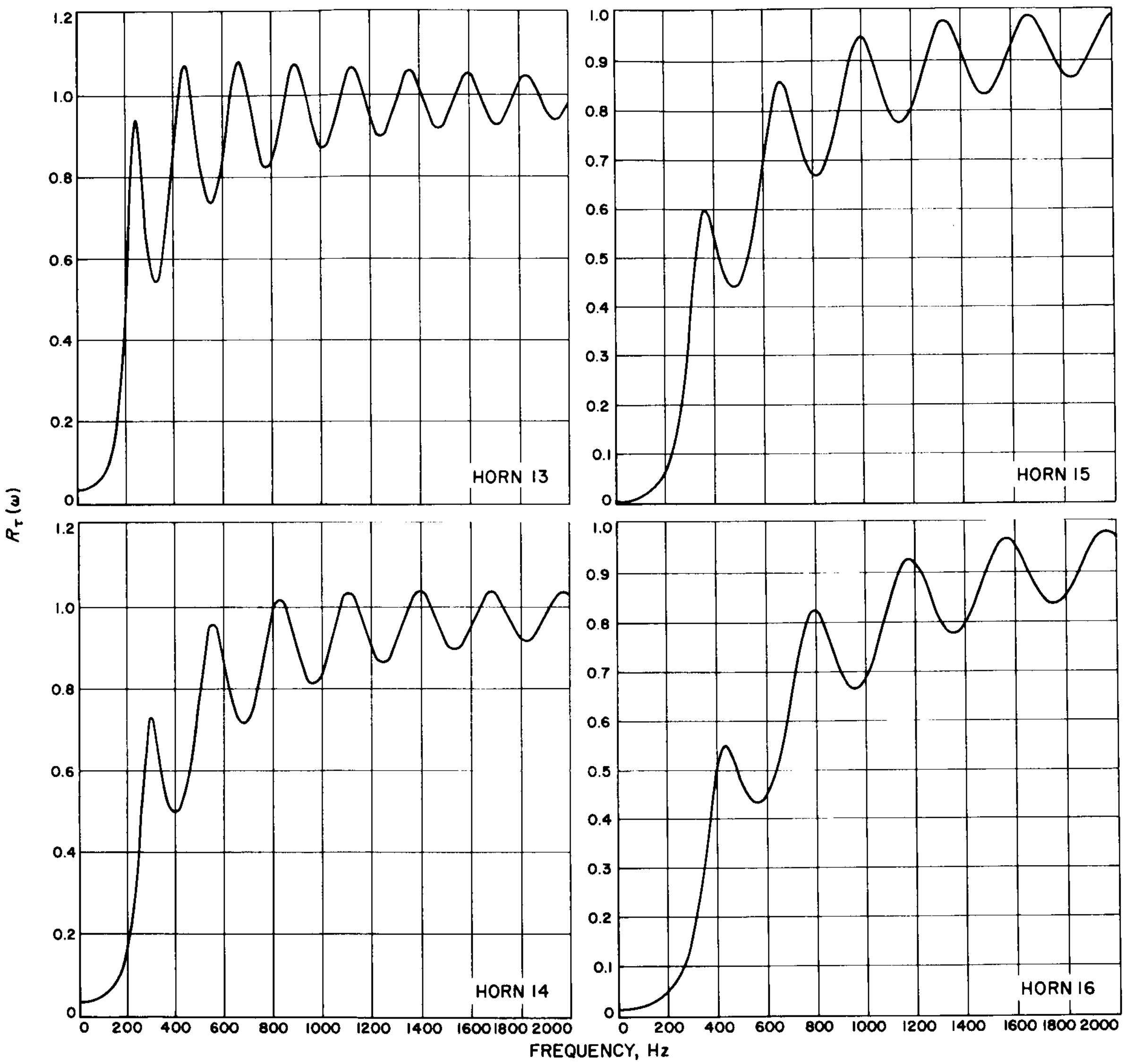

HORN 9
HORN 11
HORN 10
HORN 12
$R_T(\omega)$
FREQUENCY, Hz
0
200
400
600
800
1000
1200
1400
1600
1800
2000

Fig. 2 (contd)



HORN 13
HORN 14
HORN 15
HORN 16
$R_T(\omega)$
FREQUENCY, Hz
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.2
200
400
600
800
1000
1200
1400
1600
1800
2000

Fig. 2 (contd)

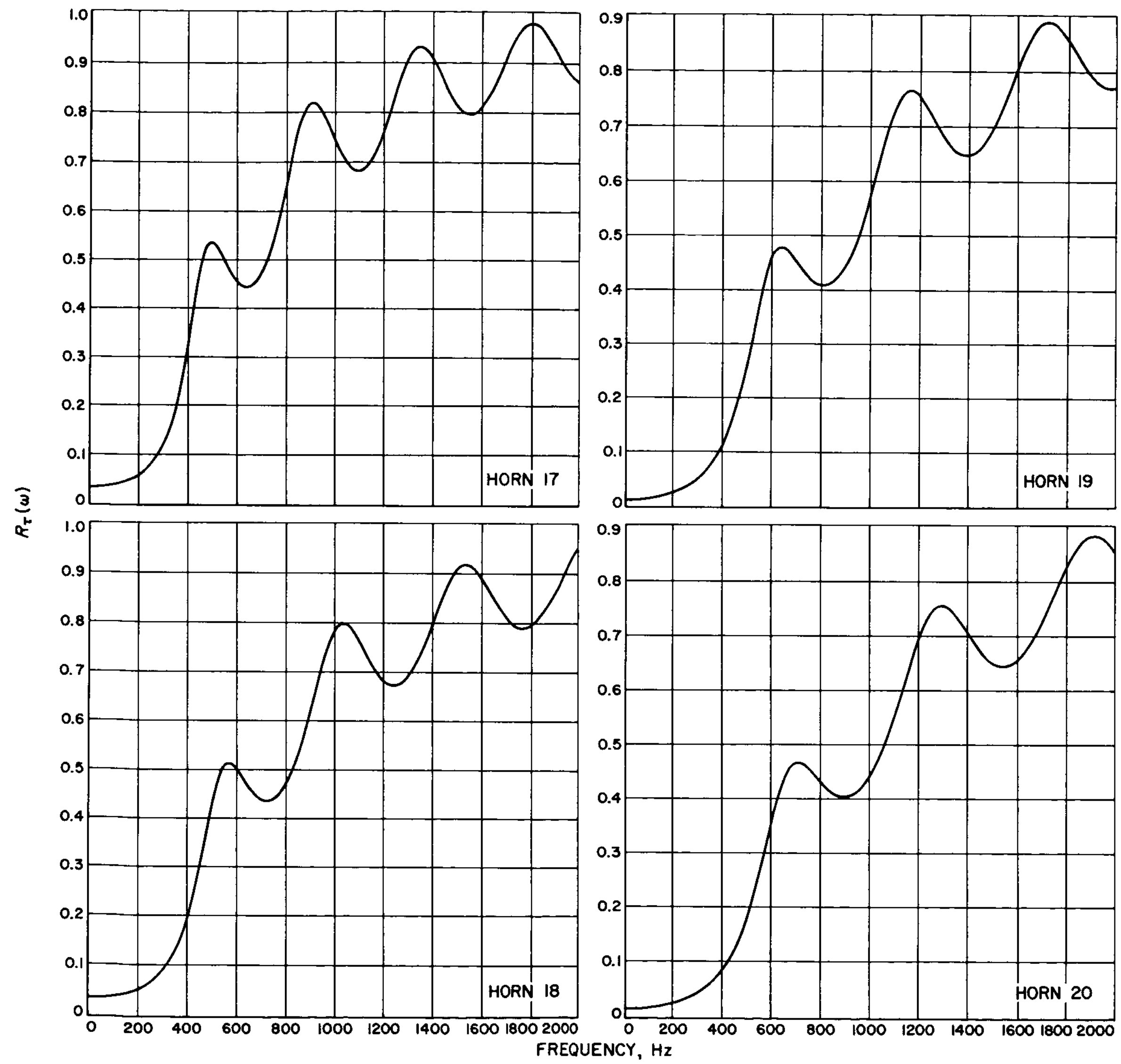

HORN 17
HORN 19
HORN 18
HORN 20
$R_T(\omega)$
FREQUENCY, Hz
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
0 200 400 600 800 1000 1200 1400 1600 1800 2000

Fig. 2 (contd)

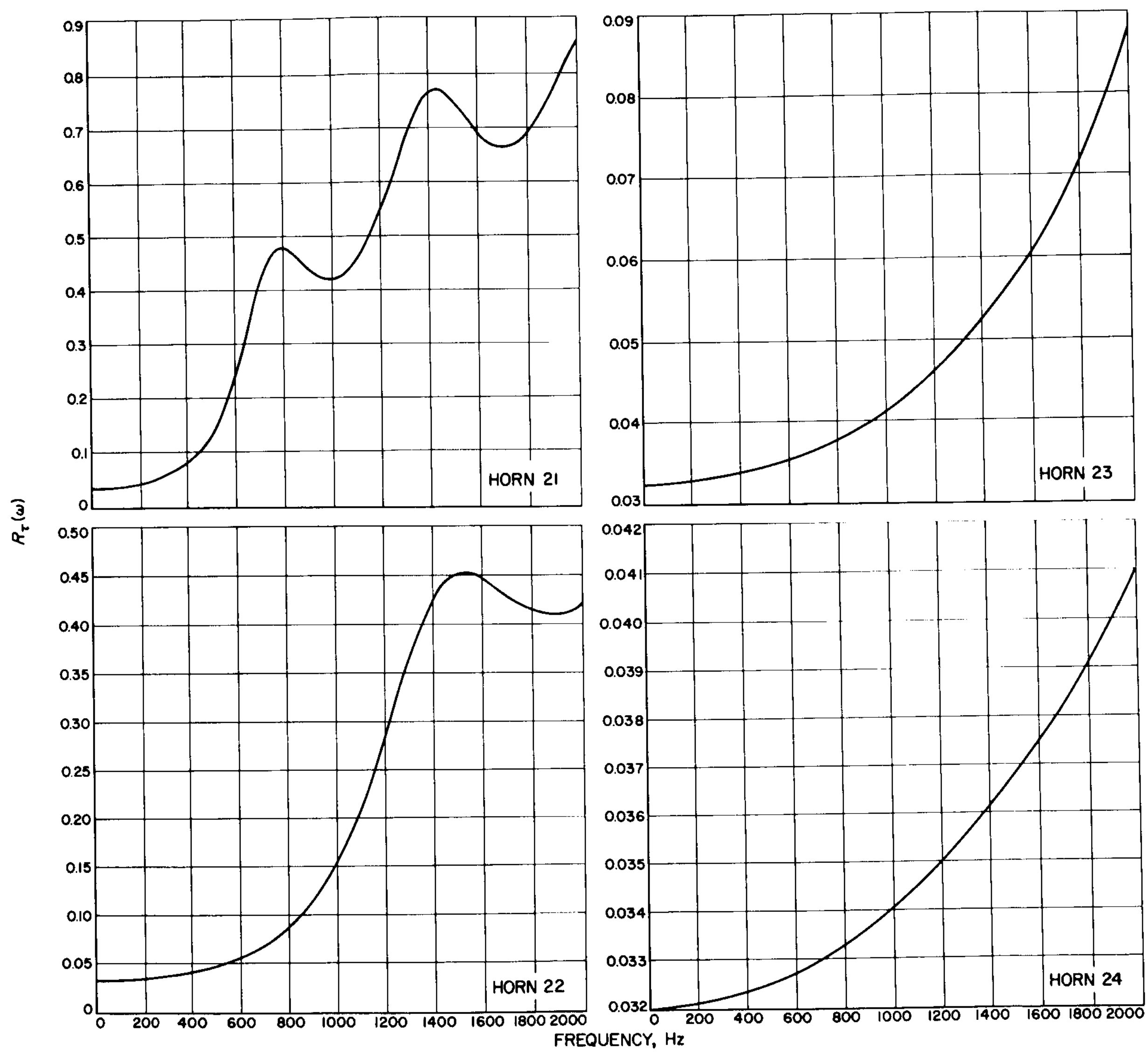


Fig. 2 (contd)

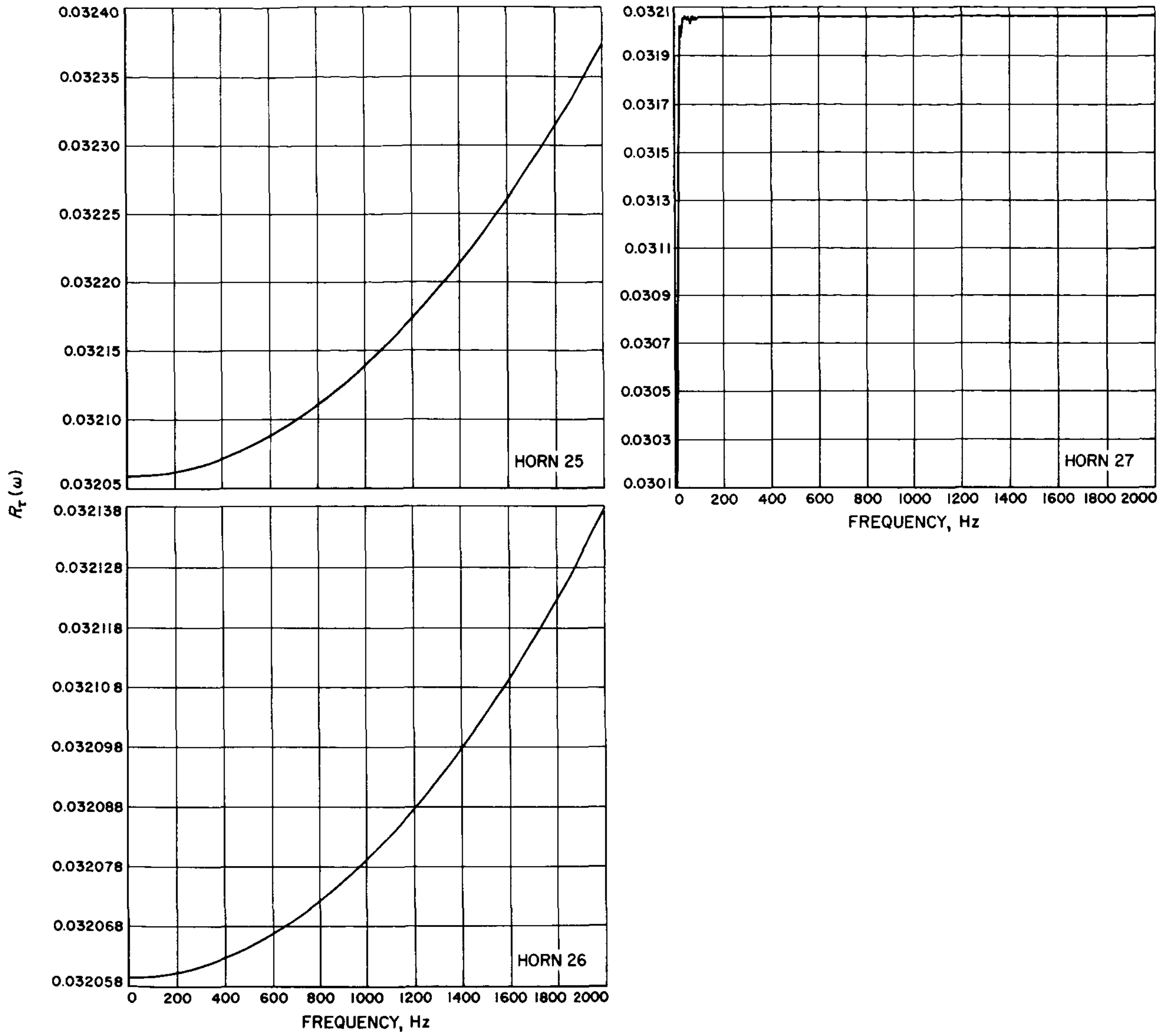


Fig. 2 (contd)

A study of the graphic responses in Fig. 2 reveals a resonant response characteristic that is just above the cutoff frequency f_c which varies with T. With this information available, acoustic horns can be designed to use this resonance characteristic. For example, horns with large resonance characteristics ($T << 1$) can be used to assist in simulating acoustic energy spectrums produced by the engines of spacecraft launch vehicles. Non-dimensionalized curves of rocket engine noise spectrums are contained in Ref. 3 and in other discussions on rocket noise.

IV. Design of Multiple Horn Arrays

A multiple horn array can be designed with the desired resonance response characteristics of a single horn and with the additional advantage from the averaging effect of several similar, but not identical, horns. This averaging technique consists of shifting the cutoff frequencies of the various horns so that the response peaks of some horns occur at the response nulls of the remaining horns. This effect tends to neutralize the extreme peak-to-null frequency spectrum oscillations.

The individual and average frequency response characteristics of four multiple horn arrays, using the horn design data listed in Table 2, are graphically portrayed in Figs. 3–6, respectively. An overlay example of the response characteristics for one array is provided as part of Fig. 3. The heavy line in the figure shows the averaging effect resulting from the different horn responses with

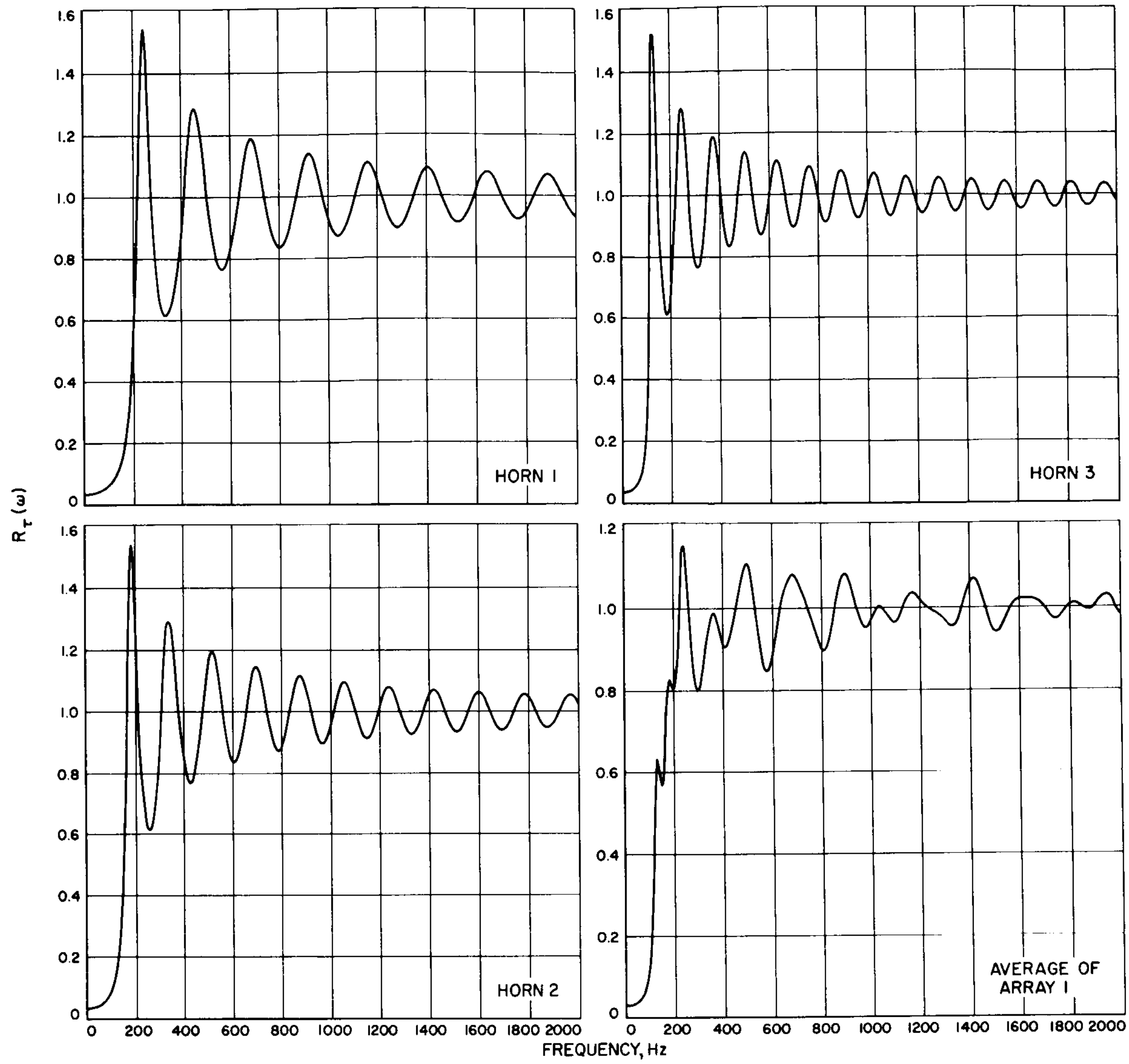


Fig. 3. Individual and average frequency response characteristics of the horns of array 1

peaks (resonances) and nulls (relative antiresonances) at different frequencies.

V. Results

This acoustic horn analysis indicates that the response characteristics of a horn can be greatly improved in the frequency range just above cutoff by carefully controlling the design parameters. This resonance response can vary over a large range and consists of several narrow spikes which would result in a very rapidly changing narrow band analysis. This narrow spike condition can be minimized by the use of multiple horn arrays. Each horn of these arrays must be carefully designed so that the transfer function of the array will provide the optimum response characteristics.

VI. Conclusions

The analysis performed in this study indicates that the concept of designing single horns and multiple horn arrays to achieve specialized response characteristics shows real promise. The analysis assumes linear acoustic response, however, this assumption becomes less valid as the acoustic levels become very large (above 135 dB in reference to 0.0002 μbar). These non-linear effects result in harmonic generation which, in turn, affects the spectrums at the higher frequencies. However, these nonlinear effects have little affect on the resonance peak of

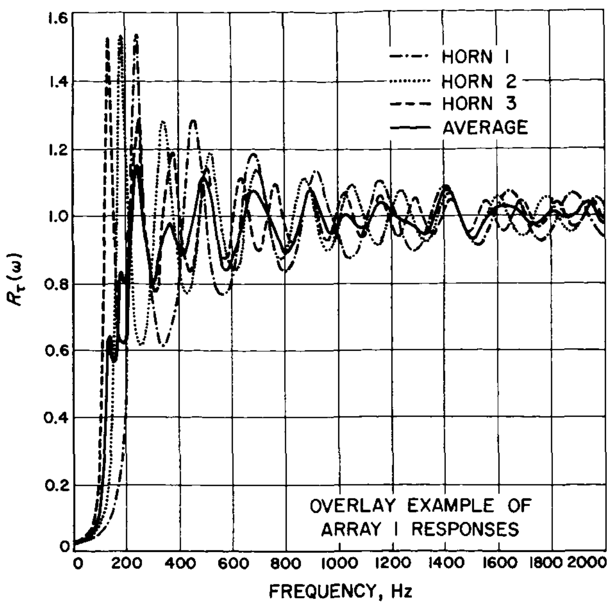


Fig. 3 (contd)

the acoustic spectrum that is transmitted as a result of the horn response characteristics. Also, the horn analysis does not allow for the damping effects of either the horn or the transmitting medium (air). The effect of such damping would decrease the peak amplitudes of the horn response function, and therefore, would decrease the response functions peak-to-null ratio. Introduction of damping could also tend to put a small frequency shift in the response function.

The choice of the parameters for each of the horns of the multiple horn arrays is somewhat arbitrary, and is mainly intended to indicate the general concept of these arrays. Future investigations into this concept will include developing techniques to determine the design parameters for each of the horns in the array so that their overall average response function is optimized. The investigation will include obtaining the desired resonance response function with the smoothest energy spectrum envelope possible.

Future investigations also include verifying these spectrum shaping techniques using a Low-Frequency Plane-Wave Sound Generator and Impedance-Measuring Device, designed and built for this same supporting research and advance development task. A report on this device is being written and will be published upon final checkout of the unit.

Table 2. Design data for multiple horn arrays

Array	Horn	T	f_c, Hz	L, m
1	1	1.0	133.3	0.7065
	2	1.0	100.0	0.9417
	3	1.0	72.5	1.2989
2	1	1.0	133.3	0.7065
	2	1.0	100.0	0.9417
	3	0.5	72.5	1.5116
3	1	1.0	100.0	0.9417
	2	1.0	117.0	0.8049
	3	1.0	134.0	0.7028
	4	1.0	151.0	0.6237
4	1	1.0	100.0	0.9417
	2	1.0	114.0	0.8261
	3	1.0	127.0	0.7415
	4	1.0	140.0	0.6727
	5	1.0	153.0	0.6155

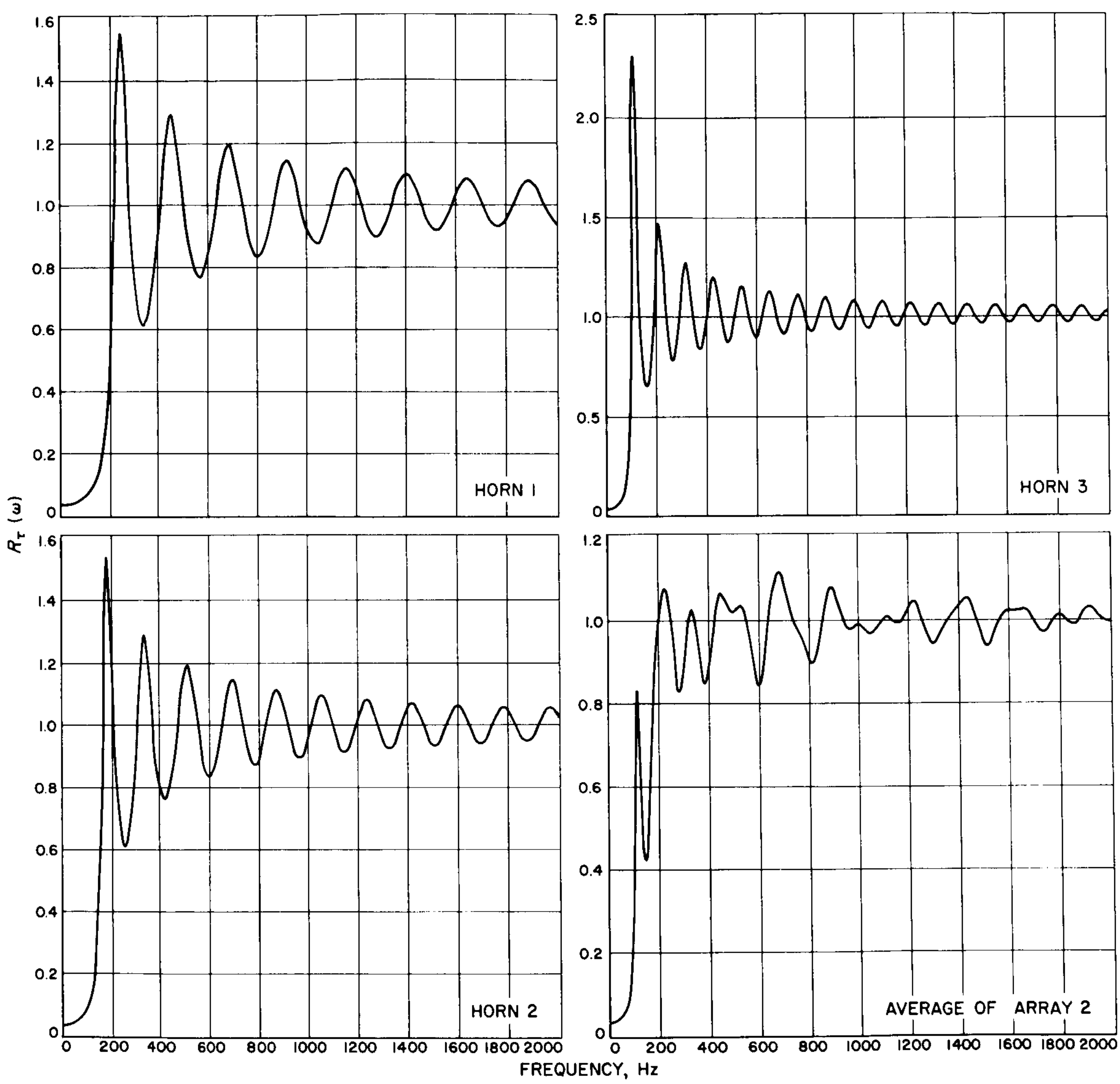


Fig. 4. Individual and average frequency response characteristics of the horns of array 2

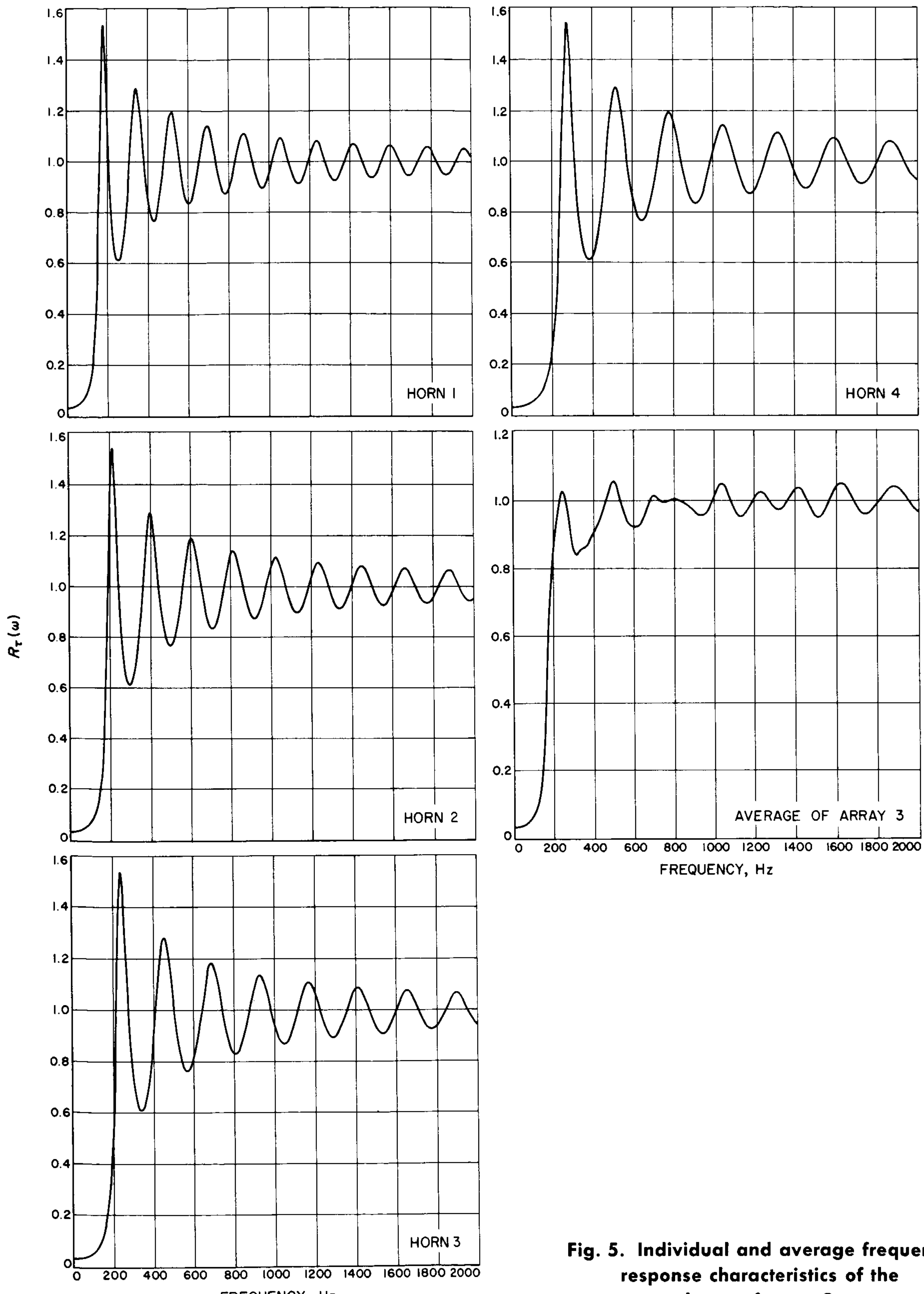


Fig. 5. Individual and average frequency response characteristics of the horns of array 3

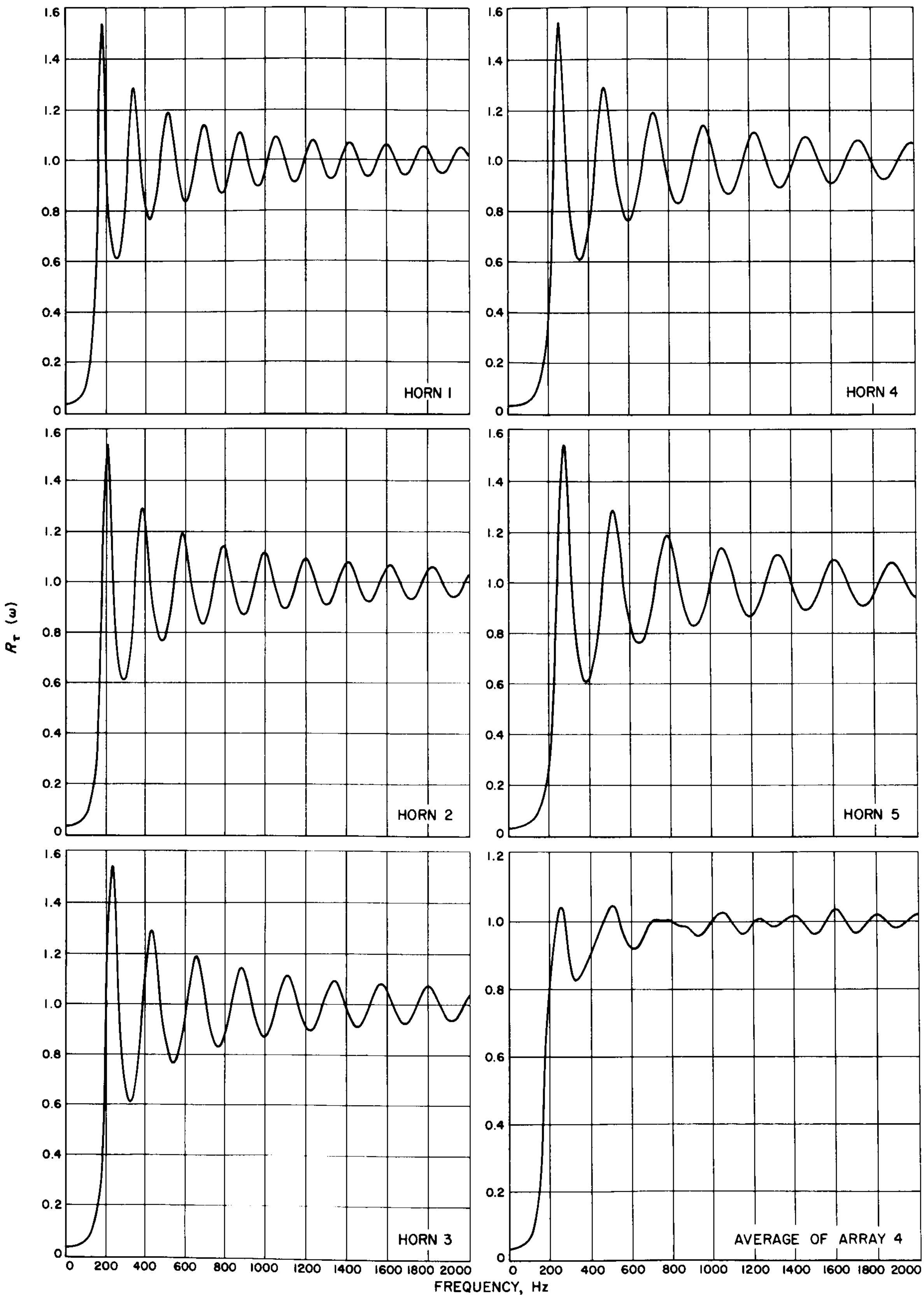


Fig. 6. Individual and average frequency response characteristics of the horns of array 4

Appendix A
Derivation of Horn Response Equations

I. Derivations

In this paper the term hyperbolic horn is used to designate those horns whose area law is given by (Refs. 1, 2, and 4):

$$S_x = S_T\,[\cosh(Nx) + T\sinh(Nx)]^2 \tag{A-1}$$

The result for the expression of the impedance at the throat z_T of a hyperbolic horn as a function of the impedance at the mouth z_M is given as follows (Ref. 1):

$$z_T = \frac{1}{T}\left(\frac{\left[\left(\frac{jk - NTz_M}{b}\right) + \left(\frac{jkT - Nz_M}{b}\right)\tanh(NL)\right]\sin(bL) + z_M\,[1 + T\tanh(NL)]\cos(bL)}{\left[\frac{1}{b}\left\{N - \left(\frac{b^2 + N^2T^2}{jkT}\right)z_M\right\} + \left(\frac{NT + jkz_M}{b}\right)\tanh(NL)\right]\sin(bL) + \left[\frac{1}{T} + \left\{1 - \frac{N(1 - T^2)\,z_M}{jkT}\right\}\tanh(NL)\right]\cos(bL)}\right) \tag{A-2}$$

This equation is valid above cutoff, that is, for $k^2 > N^2\,(f > f_c)$.

Equation (A-2) is rearranged to arrive at expressions for the at-cutoff and below-cutoff frequencies of the general hyperbolic horn through derivations Nos. 1 and 2, respectively. These expressions are further simplified for the above-cutoff, at-cutoff, and below-cutoff frequencies of the exponential horn through derivations Nos. 2, 3, and 4, respectively. Derivations Nos. 5 and 6 are alternate methods of arriving at the at-cutoff and below-cutoff expressions for the exponential horn.

A. Derivation No. 1

At the cutoff frequency, the expression for $z_T \rightarrow (0 + j0)/(0 + j0)$ is an indeterminate form, therefore, L'Hospital Rule must be used in determining the value for this expression, as shown by the following derivation.

Given Eq. (A-2) at $b = 0$ (cutoff) and $z_M \rightarrow (0 + j0)/(0 + j0)$, thus, by L'Hospital Rule and with $k = N$:

$$(z_T)_{b=0} = \frac{1}{T}\left(\frac{\frac{d}{db}\left[\left\{\left(\frac{jk - NTz_M}{b}\right) + \left(\frac{jkT - Nz_M}{b}\right)\tanh(NL)\right\}\sin(bL) + z_M\,\{1 + T\tanh(NL)\}\cos(bL)\right]}{\frac{d}{db}\left[\frac{1}{b}\left\{N - \left(\frac{b^2 + N^2T^2}{jkT}\right)\right\}z_M + \left\{\left(\frac{NT + jkz_M}{b}\right)\tanh(NL)\right\}\sin(bL) + \left\{\frac{1}{T} + \left[1 - \frac{N(1 - T^2)\,z_M}{jkT}\right]\tanh(NL)\right\}\cos(bL)\right]}\right)_{b=0} \tag{A-3}$$

$$(z_T)_{b=0} = \frac{1}{T}\left\{\frac{L\left[(jk - NTz_M) + (jkT - Nz_M)\tanh(NL)\right]\cos(bL)}{L\left[N - \left\{\frac{b^2 + N^2T^2}{jkT}\right\} z_M\right] + \left[(NT + jkz_M)\tanh(NL)\right]\cos(bL) + \sin(bL)\left[\frac{-2bz_M}{jkT}\right]}\right.$$

$$\left.\frac{+\left[z_M\{1 + T\tanh(NL)\}\cos(bL) - bLz_M\{\sin(bL)\} - (bLz_M)\{T\tanh(NL)\sin(bL)\}\right]}{+\left[\frac{1}{T} + \left\{1 - \frac{N(1-T^2)z_M}{jkT}\right\}\tanh(NL)\right]\cos(bL) - L\left[\frac{1}{T} + \left\{1 - \frac{N(1-T^2)z_M}{jkT}\right\}\tanh(NL)\right]\sin(bL)}\right\}_{b=0} \quad \text{(A-4a)}$$

$$(z_T)_{b=0} = \frac{1}{T}\left\{\frac{L\left[(jk - NTz_M) + (jkT - Nz_M)\tanh(NL)\right] + z_M\left[1 + T\tanh(NL)\right]}{L\left[\left(N - \frac{N^2T^2z_M}{jkT}\right) + (NT + jkz_M)\tanh(NL)\right] + \left[\frac{1}{T} + \left\{1 - \frac{N(1-T^2)z_M}{jkT}\right\}\tanh(NL)\right]}\right\} \quad \text{(A-4b)}$$

$$(z_T)_{b=0} = k\left\{\frac{L\left[(jk - NTz_M) + (jkT - Nz_M)\right]\tanh(NL) + z_M\left[1 + T\tanh(NL)\right]}{L\left[N(kT + jT^2Nz_M) + kT(NT + jkz_M)\tanh(NL)\right] + \left[k + \{kT + jN(1-T^2)\}\tanh(NL)\right]}\right\} \quad \text{(A-4c)}$$

$$(z_T)_{b=0} = k\left\{\frac{\left[-LNz_M\{T + \tanh(NL)\} + z_M\{1 + T\tanh(NL)\}\right] + j\left[kL\{1 + T\tanh(NL)\}\right]}{\left[LNkT\{1 + T\tanh(NL)\} + k\{1 + T\tanh(NL)\}\right] + j\left[LN^2T^2z_M + \{Lk^2Tz_M + N(1-T^2)\}\tanh(NL)\right]}\right\} \quad \text{(A-4d)}$$

Therefore, at $f = f_c$:

$$z_T = N\left\{\frac{\left[-LNz_M\{T + \tanh(NL)\} + z_M\{1 + T\tanh(NL)\}\right] + j\left[NL\{1 + T\tanh(NL)\}\right]}{\left[LN^2T\{1 + T\tanh(NL)\} + N\{1 + T\tanh(NL)\}\right] + j\left[LN^2T^2z_M + \{LN^2Tz_M + N(1-T^2)\}\tanh(NL)\right]}\right\} \quad \text{(A-5)}$$

B. Derivation No. 2

Below the cutoff frequency, the following derivation is necessary, since for $k^2 < N^2$, $b = j(|b|^2)^{1/2}$, and the various terms, such as $\cos(bL)$, need to be transformed to usable functions.

Given:

$$z_T = k \times$$

$$\left\{\frac{\left[\{(jk - NTz_M) + (jkT - Nz_M)\tanh(NL)\}\sin(bL) + bz_M\{1 + T\tanh(NL)\}\cos(bL)\right]}{\left[\{NkT + j(b + NT)z_M\} + \{(NkT^2 + jkTz_M)\tanh(NL)\}\sin(bL) + b\{k + [kT + jN(1-T^2)z_M]\}\tanh(NL)\cos(bL)\right]}\right\} \quad \text{(A-6)}$$

NOTE: Eq. (A-6) is a slightly rearranged version of Eq. (A-2).

and

$$b \equiv jB \text{ for } k < N \qquad jB = (k^2 - N^2)^{1/2} \text{ or } B^2 = N^2 - k^2 \qquad \sin(jB) = j\sinh B \qquad \cos(jB) = \cosh B$$

thus

$$z_T = k\left\{\frac{\{(jk - NTz_M) + (jkT - Nz_M)\tanh(NL)\}\,j\sinh(BL)}{\{NkT + j(-B^2 + N^2T^2)z_M\} + \{(NkT^2 + jk^2Tz_M)\tanh(NL)\}\,j\sinh(BL)}\right.$$

$$\left.\frac{+\,jBz_M\{1 + T\tanh(NL)\}\cosh(BL)}{+\,jB\{k + [kT + jN(1-T^2)z_M]\tanh(NL)\}\cosh(BL)}\right\} \quad \text{(A-7)}$$

or

$$z_T = k\left\{\frac{[k\{1 + T\tanh(NL)\}\sinh(BL)]}{[\{(N^2T^2 - B^2)\,z_M + k^2Tz_M\tanh(NL)\}\sinh(BL) + \{BN(1 - T^2)\,z_M\tanh(NL)\cosh(BL)\}]}\right.$$

$$\left.\frac{+\,j\,[Nz_M\{T + \tanh(NL)\}\sinh(BL) - Bz_M\{1 + T\tanh(NL)\}\cosh(BL)]}{-\,j\,[NkT\{1 + T\tanh(NL)\}\sinh(BL) + Bk\{1 + T\tanh(NL)\}\cosh(BL)]}\right\} \tag{A-8}$$

Therefore, for $f < f_c$:

$$z_T = k\left\{\frac{[k\{1 + T\tanh(NL)\}\sinh(BL)]}{[\{(-B^2 + N^2T^2)\,z_M + k^2Tz_M\tanh(NL)\}\sinh(BL) + \{BN(1 - T^2)\,z_M\tanh(NL)\}\cosh(BL)]}\right.$$

$$\left.\frac{+\,j\,[(Nz_M)\{T + \tanh(NL)\}\sinh(BL) - (Bz_M)\{1 + T\tanh(NL)\}\cosh(BL)]}{-\,j\,[NkT\{1 + T\tanh(NL)\}\sinh(BL) + Bk\{1 + T\tanh(NL)\}\cosh(BL)]}\right\} \tag{A-9}$$

The expression of the impedance at the throat z_T for an exponential horn ($T = 1$), in terms of the impedance at the mouth z_M, is given (Ref. 2) as follows (above cutoff):

$$Z_T = \left(\frac{\rho c}{S_1}\right)\left\{\frac{S_2Z_M[\cos(bL + \theta)] + j(\rho c)\{\sin(bL)\}}{jS_2Z_M[\sin(bL)] + (\rho c)[\cos(bL - \theta)]}\right\} \tag{A-10}$$

assuming

$$z_T = \frac{Z_T}{(\rho c/S_1)} \qquad z_M = \frac{Z_M}{(\rho c/S_2)}$$

Therefore,

$$z_T = \left\{\frac{z_M[\cos(bL + \theta)] + j[\sin(bL)]}{jz_M[\sin(bL)] + [\cos(bL - \theta)]}\right\} \tag{A-11}$$

C. Derivation No. 3

Since at the cutoff frequency $z_T \rightarrow (0 + j0)/(0 + j0)$, L'Hospital Rule is applied for the following result.

Given Eq. (A-11) at the cutoff frequency f_c (from $4k^2 = M^2$) with

$$b = \tfrac{1}{2}(4k^2 - M^2)^{1/2} = (k^2 - N^2)^{1/2} = 0 \qquad \theta = \tan^{-1}(N/b) = \pi/2$$

Therefore, at $f = f_c$:

$$z_T = \left\{\frac{z_M[\cos(\pi/2)] + j[\sin(0)]}{jz_M[\sin(0)] + [\cos(-\pi/2)]}\right\} \rightarrow \frac{0 + j0}{0 + j0} \tag{A-12}$$

Hence, by L'Hospital Rule, z_T (at $f = f_c$) is found as follows:

$$z_T = \left[\frac{(d/db)\{(z_M)\,[\cos(bL+\theta)] + j\,[\sin(bL)]\}}{(d/db)\{jz_M\,[\sin(bL)] + [\cos(bL-\theta)]\}}\right]_{b=0} \tag{A-13a}$$

$$= \left[\frac{(d/db)\{z_M\,[\cos(bL+\tan^{-1}N/b)] + j\,[\sin(bL)]\}}{(d/db)\{jz_M\,[\sin(bL)] + [\cos(bL-\tan^{-1}N/b)]\}}\right]_{b=0} \tag{A-13b}$$

$$= \left\{\frac{-z_M\,[\sin(bL+\tan^{-1}N/b)]\left[L + \dfrac{(-N/b^2)}{(1+N^2/b^2)}\right] + j\,(L)\,[\cos(bL)]}{j\,(z_M L)\,[\cos(bL)] - [\sin(bL-\tan^{-1}N/b)]\left[L + \dfrac{(N/b^2)}{(1+N^2/b^2)}\right]}\right\}_{b=0} \tag{A-13c}$$

$$= \left\{\frac{-z_M\,[\sin(bL+\tan^{-1}N/b)]\left[\dfrac{L\,(N^2+b^2)-N}{(N^2+b^2)}\right] + j\,(L)\,[\cos(bL)]}{j\,(z_M L)\,[\cos(bL)] - [\sin(bL-\tan^{-1}N/b)]\left[\dfrac{L\,(N^2+b^2)+N}{(N^2+b^2)}\right]}\right\}_{b=0} \tag{A-13d}$$

$$= \frac{-(z_M)\,[\sin(\pi/2)]\left(\dfrac{LN^2-N}{N^2}\right) + j\,(L)\,[\cos(0)]}{j\,(z_M L)\,[\cos(0)] - \sin(-\pi/2)\left(\dfrac{LN^2+N}{N^2}\right)} \tag{A-13e}$$

$$= \frac{-(z_M)\left(L - \dfrac{1}{N}\right) + j\,(L)}{j\,(z_M L) + \left(L + \dfrac{1}{N}\right)} = \frac{+(z_M)\left(\dfrac{1}{M/2} - L\right) + j\,(L)}{j\,(z_M L) + \left(L + \dfrac{1}{M/2}\right)} = \frac{z_M\left(1 - \dfrac{ML}{2}\right) + j\left(\dfrac{ML}{2}\right)}{jz_M\left(\dfrac{LM}{2}\right) + \left(\dfrac{ML}{2} + 1\right)} \tag{A-13f}$$

Therefore, at $f = f_c$:

$$z_T = \frac{z_M\left(1 - \dfrac{LM}{2}\right) + j\left(\dfrac{ML}{2}\right)}{\left(1 + \dfrac{ML}{2}\right) + jz_M\left(\dfrac{LM}{2}\right)} \tag{A-14}$$

D. Derivation No. 4

Below the cutoff frequency, that is for $2k < M$ (or $k < N$), following results are derived:

Given Eq. (A-11) and the following definitions of trigonometric identities:

(a) $b = jB$ (b) $\cos(\theta \pm jB) = \cos(\theta)\cosh(B) \mp j\sin(\theta)\sinh(B)$

(c) $\sin(jB) = j\sinh B$ (d) $\cos(jB) = \cosh B$

(e) $\cos(\pm\theta + bL) = \cos(\pm\theta + jBL) = \cos(\theta)\cosh(BL) \mp j\sin(\theta)\sinh(B)$

(f) $\theta = \tan^{-1}(N/b) = \tan^{-1}(N/jB)$ (g) $\cos(\theta) = jB/(N^2 - B^2)^{1/2}$ (h) $\sin(\theta) = N/(N^2 - B^2)^{1/2}$

then

$$z_T = \left\{ \frac{z_M [\cos(\theta) \cosh(BL) - j \sin(\theta) \sinh(BL)] + j [j \sinh(BL)]}{j z_M [j \sinh(BL)] + [\cos(\theta) \cosh(BL) + j \{\sin(\theta) \sinh(BL)\}]} \right\} \tag{A-15}$$

Therefore, for $f < f_c$:

$$z_T = \left\{ \frac{\left[\sinh(BL) - j \dfrac{z_M}{(N^2 - B^2)^{1/2}} \{B \cosh(BL) - N \sinh(BL)\}\right]}{\left[z_M \sinh(BL) - j \dfrac{1}{(N^2 - B^2)^{1/2}} \{B \cosh(BL) + N \sinh(BL)\}\right]} \right\} \tag{A-16}$$

E. Derivation No. 5

Equation (A-14) can also be derived from the general expression for z_T using the family parameter T equal to 1. Given Eq. (A-5) with $T = 1$ for the exponential horn, then Eq. (A-5) reduces to:

$$z_T = k \left\{ \frac{[-LN z_M \{1 + \tanh(NL)\} + z_M \{1 + \tanh(NL)\}] + j [NL \{1 + \tanh(NL)\}]}{[LN^2 \{1 + \tanh(NL)\} + N \{1 + \tanh(NL)\}] + j [LN^2 z_M \{1 + \tanh(NL)\}]} \right\} \tag{A-17}$$

$$z_T = \frac{z_M (1 - LN) + j (NL)}{(1 + LN) + j z_M (LN)} \tag{A-18}$$

however, $N = M/2$; therefore,

$$z_T = \frac{z_M \left(1 - \dfrac{LM}{2}\right) + j \left(\dfrac{ML}{2}\right)}{\left(1 + \dfrac{ML}{2}\right) + j z_M \left(\dfrac{LM}{2}\right)} \equiv \text{Eq. (A-14)}$$

F. Derivation No. 6

Equation (A-16) can also be derived from the general expression for z_T using the family parameter T equal to 1. Given Eq. (A-9), then, for $T = 1$:

$$z_T = k \left\{ \frac{[k \sinh(BL)] + j [z_M \{N \sinh(BL) - B \cosh(BL)\}]}{[z_M k^2 \sinh(BL)] - j [k \{N \sinh(BL) + B \cosh(BL)\}]} \right\} \tag{A-19}$$

for $jB = b$ or $B = -jb$:

$$z_T = k \left\{ \frac{[k \sinh(BL)] + j [z_M \{N \sinh(BL) + jb \cosh(BL)\}]}{[z_M k^2 \sinh(BL)] - j [k \{N \sinh(BL) - jb \cosh(BL)\}]} \right\} \tag{A-20}$$

with $\tan^{-1}(N/b) = \theta$, then

$$z_T = k\left\{\frac{[(k/b)\sinh(BL)] + j\,[z_M\,(N/b)\sinh(BL) + j\cosh(BL)\}]}{[z_M k\,\{(k/b)\sinh(BL)\}] - j\,[k\,\{(N/b)\sinh(BL) - j\cosh(BL)\}]}\right\} \tag{A-21}$$

with $\sec\theta = k/b$, then

$$z_T = k\left\{\frac{[\sec(\theta)\sinh(BL)] + j\,[z_M\,\{\tan(\theta)\sinh(BL) + j\cosh(BL)\}]}{[z_M k\,\{\sec(\theta)\sinh(BL)\}] - j\,[k\,\{\tan(\theta)\sinh(BL) - j\cosh(BL)\}]}\right\} \tag{A-22}$$

$$z_T = k\left\{\frac{[\sinh(BL)] + j\,[z_M\,\{\sin(\theta)\sinh(BL) + j\cos(\theta)\cosh(BL)\}]}{[z_M k\,\{\sinh(BL)\}] - jk\,[\sin(\theta)\sinh(BL) - j\,\{\cos(\theta)\cosh(BL)\}]}\right\} \tag{A-23}$$

however, $\cos\theta = jB/(N^2 - B^2)^{1/2}$ and $\sin\theta = N/(N^2 - B^2)^{1/2}$; therefore,

$$z_T = \left\{\frac{[\sinh(BL) - j\,\frac{z_M}{(N^2 - B^2)^{1/2}}\,\{B\cosh(BL) - N\sinh(BL)\}]}{[z_M\sinh(BL) - j\,\frac{1}{(N^2 - B^2)^{1/2}}\,\{B\cosh(BL) + N\sinh(BL)\}]}\right\} \equiv \text{Eq. (A-16)}$$

Appendix B

Digital Computer Program

```
C                                                                       ER000300
C
C     ACOUSTICAL HORN PROBLEM  APRIL, 1966
C                                                                       XXXXXXXX
      COMPLEX T,N,L,F,B,T1,K,A,ZM,T2,T3,SINH,COSH,TANH,T4,T5,T6,T7,
     1ZT,ZT1,C,SMALLB,SINE,COSINE,T8,T9,T10, X, X2, THETA, NUM
C           --                                                          ER000200
C     THESE ARE CONSTANTS USED THROUGHOUT THE PROGRAM
      C=(344.0,0.0)
      A=(0.10,0.0)
C                                                                       ER000500
C
  101 IFIRST=0
C
C     READ IN VARIABLES - HORN NUMBER, T, N AND L.
      READ (5,1)IHORN,  T,  N,  L
    1 FORMAT  (I5, 6F10.4)
C     -- -**
C     IF HORN NUMBER IS EQUAL TO ZERO, NO MORE DATA.
      IF  (IHORN) 1000,1000,2
C
C     PRINT OUT HEADER
C
    2 WRITE  (6,4)IHORN,T,N,L
C
    4 FORMAT (1H1, 12X, 21HACOUSTIC HORN PROBLEM,    /,28X,4HHORN,
     1I2,5H T =  , 2F5.2, 5H N =  , 2F5.2, 5H L =  , 2F5.2)
C                                                                       XXXXXXXX
C     IF HORN IS ONE, TWO, OR THREE, DO FORMULAS 9, 3, AND 2
      GO TO (6,6,6,100,100),IHORN
C
C
C     SET F TO 5 CPS, GO TO EQUATION 9
    6 F=(5.0,0.0)
  402 T1=N*L
      T2=CEXP(T1)
      T3=CEXP(-T1)
      TANH=(T2-T3)/(T2+T3)
C
C                                                                       IN012800
C     IF F HAS REACHED 100 CPS, GO TO EQUATION 3
    7 FF=REAL(F)
      IF  (FF-100.0)   8,20,20
C                                                                       IN012600
C     COMPUTE B, ZM, TANH, COSH, AND SINH


    8 B=(1.83,0.0)*CSQRT((1.0,0.0)-F*F*(0.0001,0.0))
C
C     COMPUTE K
      K=  (.0183,0.0)*F
      T1=K*A
C                                                                       IN011600
      ZM=(0.24,0.0)*T1*T1+(0.0,1.0)*(0.57,0.0)*T1
      T1=B*L
      T2=CEXP(T1)
      T3=CEXP(-T1)
      SINH=(T2-T3)/(2.0,0.0)
      COSH=(T2+T3)/(2.0,0.0)
```

```
C                                                                IN011400
C
C     COMPUTE EQUATION 9
C     -- ---
    9 T1=K*(((1.0,0.0)+T*TANH)*SINH)
      T2=(N*ZM*(T+TANH)*SINH)
      T3=-B*ZM*((1.0,0.0)+T*TANH)*COSH
      T4=((-B*B+N*N*T*T)*ZM+K*K*T*ZM*TANH)*SINH
      T5=(B*N*((1.0,0.)-T*T)*ZM*TANH)*COSH
      T6=N*K*T*((1.0,0.0)+T*TANH)*SINH
      T7=B*K*((1.0,0.0)+T*TANH)*COSH
C                                                                IN009500
C                                                                IN007215
C
C                                                                IN009200
C     FORM ZT
      ZT=K*((T1+(0.0,1.0)*(T2+T3))/(T4+T5-(0.0,1.0)*(T6+T7)))
C                                                                IN007205
C     IS THIS THE FIRST TIME THROUGH
      IF (IFIRST) 14,14,16
C     YES - GO BACK, COMPUTE EQUATION WITH OTHER ZM
   14 IFIRST=1
      ZT1=ZT
      ZM=(1.0,0.0)
      GO TO 9
C                                                                IN007500
C                                                                IN007300
C     PRINT OUT BOTH VALUES WITH F
   16 FF=REAL(F)
      WRITE (6,18) FF,ZT1,ZT
   18 FORMAT (1H0, 2X, F6.1,4H CPS, 3X, 2E17.5, 3X, 2E17.5)
C     ADD 5 CPS TO F
      F=F+(5.0,0.0)
      IFIRST=0
      GO TO 7

C                                                                IN007100
C                                                                IN006900
C     -- -*-
C     COMPUTE EQUATION 3
C     FIRST RESET ZM
C     COMPUTE K
   20 K=(.0183, 0.0)*F
   21 T1=K*A
      ZM=(0.24,0.0)*T1*T1+(0.0,1.0)*(0.57,0.0)*T1
C                                                                IN003500
C                                                                IN003300
   22 T1=-L*N*ZM*(T+TANH)
      T2=ZM*((1.0,0.0)+T*TANH)
      T3=N*L*((1.0,0.0)+T*TANH)
      T4=L*N*N*T*((1.0,0.0)+T*TANH)
      T5=N*((1.0,0.0)+T*TANH)
      T6=L*N*N*T*T*ZM
      T7=(L*N*N*T*ZM+N*((1.0,0.0)-T*T))*TANH
      ZT=N*((T1+T2+(0.0,1.0)*T3)/(T4+T5+(0.0,1.0)*(T6+T7)))
```

```
C                                                                XXXXXXXX
C                                                                XXXXXXXX
C                                                                IN000800
C     SUBSTITUTE OTHER VALUE OF ZM
      IF (IFIRST)  24,24,26
   24 IFIRST=1
      ZT1=ZT
      ZM=(1.C,0.0)
      GO TO 22
C         ---                                                    IN000200
   26 FF=REAL(F)
      IFIRST=0
      WRITE (6,18)FF,ZT1,ZT
C                                                                XXXXXXXX
C     NOW COMPUTE EQUATION 2
C     DETERMINE IF F IS EQUAL TO 150 CPS
   28 FF=REAL(F)
      IF (FF-150.0)  30,32,32
C                                                                IN000300
C     F UNDER 150 CPS - ADD 2 CPS
   30 F=F+(2.C,0.0)
      GO TO 36
C
C     ADD 5 CPS TO F
   32 F=F+(5.0,0.0)
C                                                                XXXXXXXX


C     DETERMINE IF F = 500 CPS
   34 FF=REAL(F)
      IF (FF-505.0)  36,50,50
C
C
C     COMPUTE EQUATION 2   GET FIRST VALUE OF ZM
   36 K= (.0183,0.0)*F
      T1=K*A
      ZM=(0.24,0.0)*T1*T1+(0.0,1.0)*(0.57,0.0)*T1
   37 SMALLB=(1.83,0.0)*CSQRT((0.0001,0.0)*F*F-(1.0,0.0))
      SINE=CSIN(SMALLB*L)
      COSINE=CCOS(SMALLB*L)
   38 T1=(-N*T*ZM+(0.0,1.0)*K)/SMALLB
      T2=(-N*ZM+(0.0,1.0)*T*K)/SMALLB
      T3=(T1+T2*TANH)*SINE
      T4=ZM*((1.0,0.0)+T*TANH)*COSINE
      T5=(SMALLB*SMALLB+N*N*T*T)/((.0,1.0)*K*T)
      T6=(N-T5*ZM)/SMALLB
      T7=(N*T+(0.0,1.0)*K*ZM)/SMALLB
      T8=(T6+T7*TANH)*SINE
      T9=(1.C,0.0)-(N*((1.0,0.0)-T*T)*ZM)/((0.0,1.0)*K*T)
      T10=((1.0,0.0)/T+T9*TANH)*COSINE
      ZT=(T3+T4)/(T*(T8+T10))
C
C
C     GO COMPUTE EQUATION AGAIN FOR SECOND ZM
      IF (IFIRST) 42,42,44
   42 IFIRST=1
      ZM=(1.0,0.0)
      ZT1=ZT
      GO TO 38
```

```
   44 FF=REAL(F)
      WRITE (6,18)FF,ZT1,ZT
      IFIRST=0
      GO TO 28
C     --- -**
C
C     IF THIS IS HORN 3, GO THROUGH 2ND SET OF EQUATIONS
   50 GO TO (101,101,51),IHORN
C     PRINT SEPARATOR
   51 WRITE(6,52)
   52 FORMAT(50H ..................................................)
C
C**********************************************************************
C
C     COMPUTE EQUATION 22 - SPECIAL HYPERBOLIC HORN
      IFIRST=0

C     FIRST SET F TO 5 CPS
      F=(5.0,0.0)
C     COMPUTE ZM
   53 K=(0.0183,0.0)*F
      T1=K*A
      ZM=(0.24,0.0)*T1*T1+(0.0,1.0)*(0.57,0.0)*T1
C
C     COMPUTE B
      B=(1.83,0.0)*CSQRT((1.0,0.0)-(0.0001,0.0)*F*F)
      T1=CEXP(B*L)
      T2=CEXP(-B*L)
C     FORM SINH(BL) AND COSH(BL)
      T6=(T1+T2)/(2.0,0.0)
      T7=(T1-T2)/(2.0,0.0)
      CC=REAL((B+N)/(B-N))
      CC=ALOG(CC*CC)/4.0
      T1=CMPLX(CC,0.0)
      T8=CEXP(T1)
      T9=CEXP(-T1)
C     FORM COSH AND SINH OF THE LN
      T2=(T8+T9)/(2.0,0.0)
      T3= (T8-T9)/(2.0,0.0)
   54 ZT=(ZM*(T2*T6-T3*T7)-T7)/(-ZM*T7+T2*T6+T3*T7)
C                                                                XXXXXXXX
C     IS THIS THE FIRST TIME WITH THIS CPS
      IF (IFIRST) 62,62,64
C     FIRST TIME - SAVE ZT
   62 ZT1=ZT
      ZM=(1.0,0.0)
      IFIRST=1
      GO TO 54
   64 FF=REAL(F)
      WRITE  (6,18)  FF,ZT1,ZT
C
C     DETERMINE IF CUT OFF HAS BEEN REACHED.
      F=F+(5.0 ,0.0)
      IFIRST=0
      FF=REAL(F)
      IF (FF-100.0)  53,66,66
C
C     COMPUTE EQUATION 17
   66 K=(.0183,0.0)*F
      T1=K*A
      ZM=(0.24,0.0)*T1*T1+(0.0,1.0)*(0.57,0.0)*T1
      IFIRST = 0
```

Nomenclature

a radius of cross sectional area at mouth, m

b $+ \frac{1}{2}(4k^2 - M^2)^{1/2} = +(k^2 - N^2)^{1/2}$, m^{-1}

B $+(N^2 - k^2)^{1/2} = jb$, m^{-1}

c velocity of sound in medium transmitting sound waves (air), m/s

f frequency of sound waves, Hz

f_c cutoff frequency of sound waves, Hz

k wave number $= 2\pi/\lambda = \omega/c$, m^{-1}

L length of horn, m

M flare constant of exponential horn $= 2N$, m^{-1}

N flare constant of hyperbolic horn, m^{-1}

R_T from $z_T = R_T + jX_T$, N-s-m^{-5}

S_x cross sectional area of hyperbolic (and exponential) horns at a position x units from throat, m^2

S_M = cross sectional area at mouth, m^2

S_T = cross sectional area at throat, m^2

X_T from $z_T = R_T + jX_T$, N-s-m^{-5}

$z_i\,(i = M, T)$ acoustical impedances of various acoustic systems to be described, $z_i = Z_i/(\rho c/S_i)$

z_M = acoustical impedance at mouth

z_T = acoustical impedance at throat

$Z_i\,(i = M, T)$ acoustical impedances of various acoustic systems to be described (mks acoustic Ω), N-s-m^{-5}

z_M = acoustical impedance at mouth of particular horn being described, N-s-m^{-5}

z_T = acoustical impedance at throat of particular horn being described, N-s-m^{-5}

ϕ $\tan^{-1}(X_T/R_T)$, deg

References

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2. Olson, H. R., *Acoustical Engineering*, pp. 103–114. D. Van Nostrand Co., Inc., New Jersey, 1957.
3. Crandall, S., *Random Vibration*. Massachusetts Institute of Technology Press, Massachusetts, 1963.
4. Salmon, V., "A New Family of Horns," *J. Acoust. Soc. Am.*, Vol. 17, No. 3, Jan. 1946.

```
      T2=(L*(3.66,0.0))/(2.0,0.0)
   72 ZT=(ZM*((1.0,0.0)-T2)+(0.0,1.0)*T2)/((1.0,0.0)+T2+(0.0,1.0)*ZM*T2)
      IF (IFIRST) 74,74,78
   74 ZT1=ZT
      ZM=(1.0,0.0)
      IFIRST=1
      GO TO 72
   78 FF=REAL(F)
      WRITE (6,18) FF,ZT1,ZT
      IFIRST=0
C
C     COMPUTE EQUATION 16
C
C     ADD 2 CPS TO F
      F=F+(2.0,0.0)
C
C
C     COMPUTE K
  780 K=(.0183,0.0)*F
      T1=K*A
C
C     COMPUTE FIRST ZM
      ZM=(0.24,0.0)*T1*T1+(0.0,1.0)*(0.57,0.0)*T1
C
C
C     FIRST COMPUTE SMALL B
      SMALLB=(1.83,0.0)*CSQRT((.0001,0.0)*F*F-(1.0,0.0))
C     NOW COMPUTE THETA
      CC=REAL(N/SMALLB)
      CC=ATAN(CC)
      THETA=CMPLX(CC,0.0)
C
C     GOT THETA - NOW COMPUTE EQUATION 16
C
C     FIRST GET SINES AND COSINES
  787 SINE=CSIN(SMALLB*L)
      COSINE = CCOS(SMALLB*L+THETA)
      T1=CCOS(SMALLB*L-THETA)
      T2=ZM*COSINE+(0.0,1.0)*SINE
      T3=(0.0,1.0)*ZM*SINE+T1
      ZT=T2/T3
C
C     GO THRU FOR OTHER ZM
      IF (IFIRST) 788,788,789
  788 IFIRST=1
      ZT1=ZT
      ZM=(1.0,0.0)

      GO TO 787
  789 FF=REAL(F)
      IFIRST=0
      WRITE (6,18) FF,ZT1,ZT
C
C     DETERMINE IF PROBLEM IS OVER
      IF (FF-500.0) 80,101,101
   80 IF (FF-150.0) 82,86,86
   82 F=F+(2.0,0.0)
      GO TO 780
   86 F=F+(5.0,0.0)
      GO TO 780
C
  100 CONTINUE
 1000 STOP
      END
```